

# ONLINE CONTROL AND MOMENT OF INERTIA ESTIMATION OF TETHERED DEBRIS

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# Outline

- Introduction
- Methodology
  - System Model
  - Estimation
  - Control
  - Parameters
- Performance
- Conclusion



# Outline

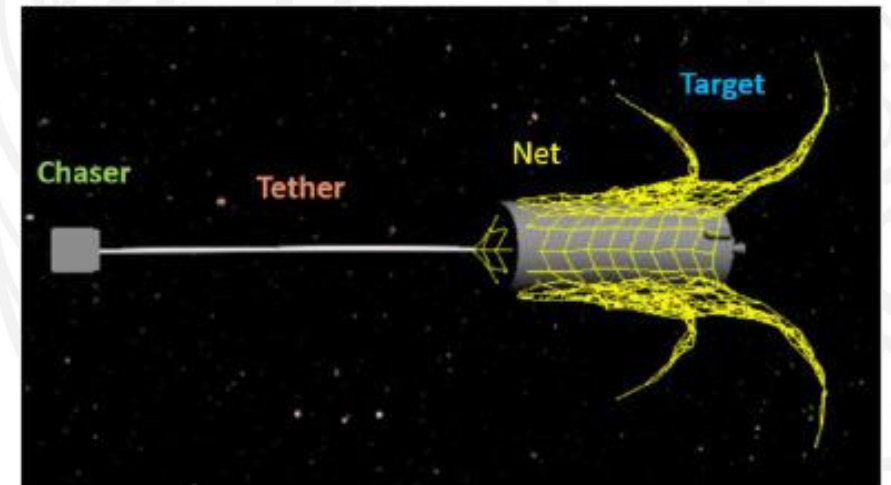
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# Introduction

## Tether-based Capture of Debris

- Tether-based capture of debris include tether nets and harpoons
  - Safer capture of debris (longer distances can be maintained)
  - More difficult to control
    - De-tumbling, model prediction, collision prevention, etc.
    - Chaotic motion, coupled dynamics
    - Unknown debris parameters and states



(Botta et al. 2020)

# Introduction

## Prior work by our lab:

- Controls (Field and Botta [1])
  - Maintain safe operations
  - Assume known debris tether attachment point, attitude, and position
- Estimation (Bourabah et. al. [2])
  - Estimate attitude, angular rate, and principal Mass Moments of Inertia (MMI)
  - Assume controls know true debris states

**This work performs estimation and control simultaneously**

**(i.e., controls don't assume known debris states)**

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# System Model

- Rigid body chaser and debris connected by massless, extensible tether modeled as a single spring-damper

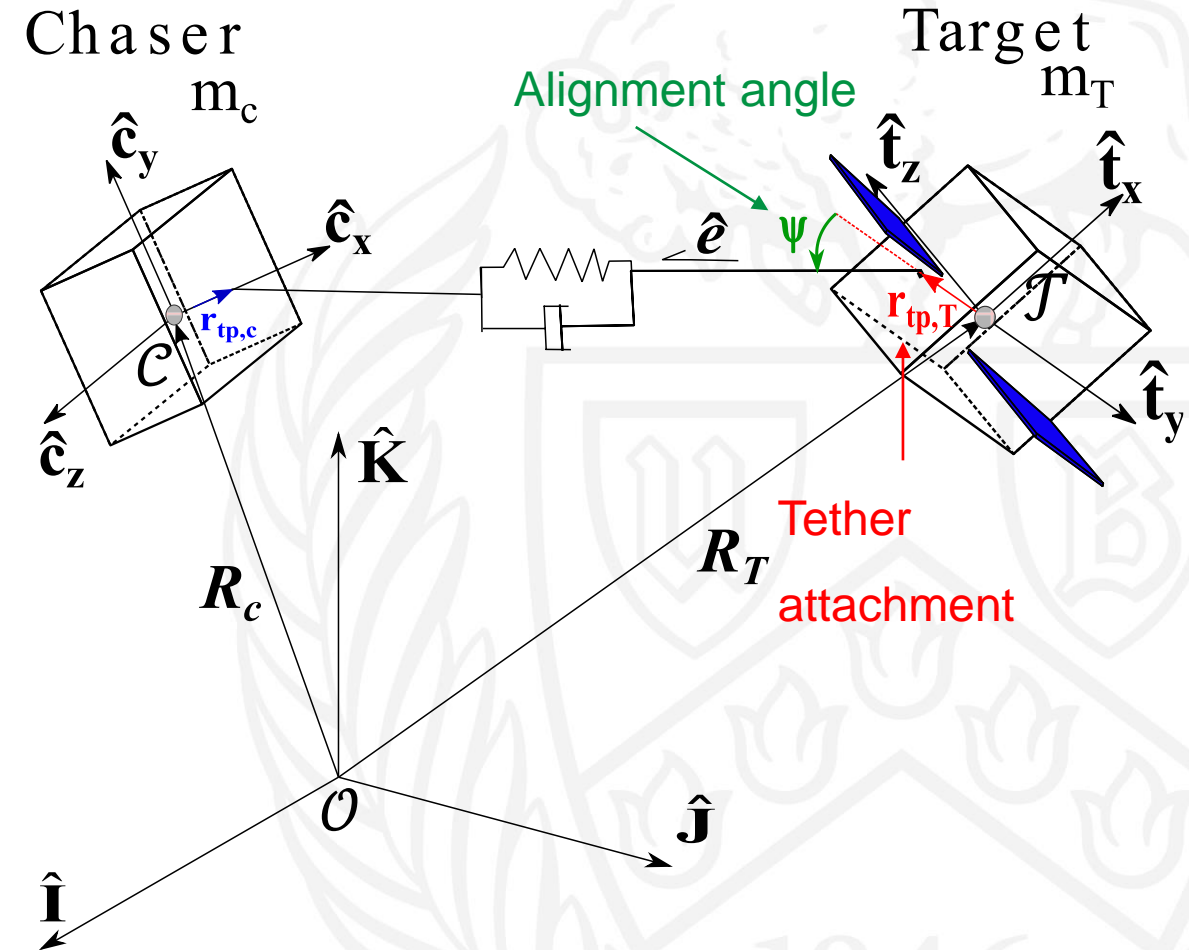
$\mathcal{O}[\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}]$ : ECI frame

$\mathcal{C}[\hat{\mathbf{c}}_x, \hat{\mathbf{c}}_y, \hat{\mathbf{c}}_z]$ : Chaser body frame

$\mathcal{T}[\hat{\mathbf{t}}_x, \hat{\mathbf{t}}_y, \hat{\mathbf{t}}_z]$ : Target body frame

$\mathbf{r}_{tp}$ : Tether attachment point

$\Psi$ : Alignment angle



# Challenges

## Controls

- Require knowledge of debris states (Attitude, angular rates)



Estimated

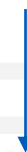
- Requires knowledge of chaser states



Assumed known

## Estimation

- Requires knowledge of acting moments on target



Debris connected via tether

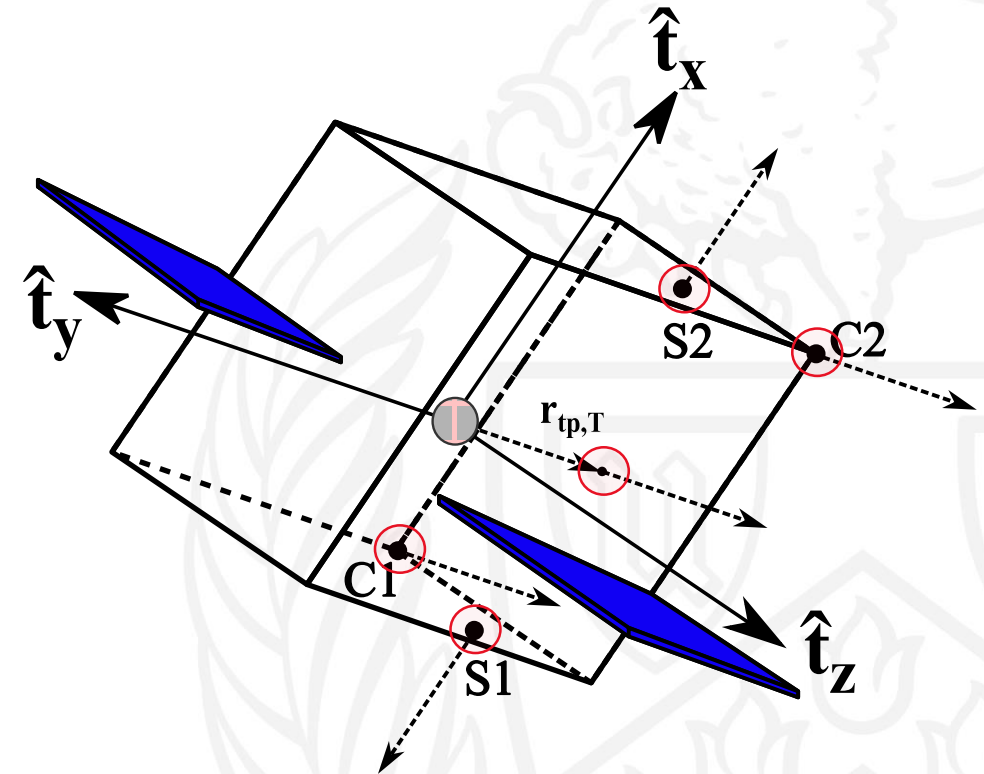
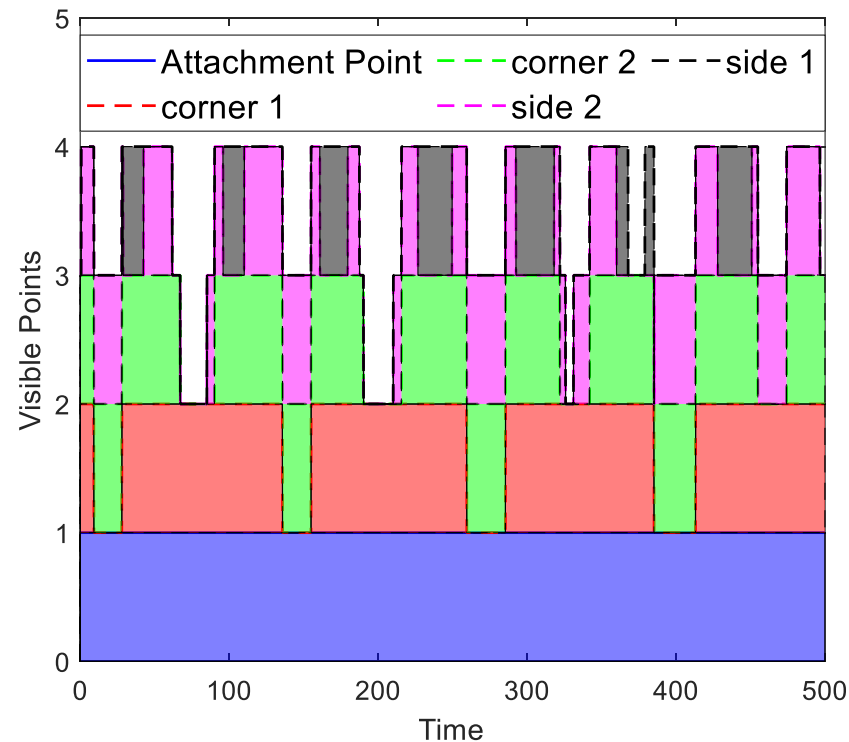


Tether tension provides moment for estimation



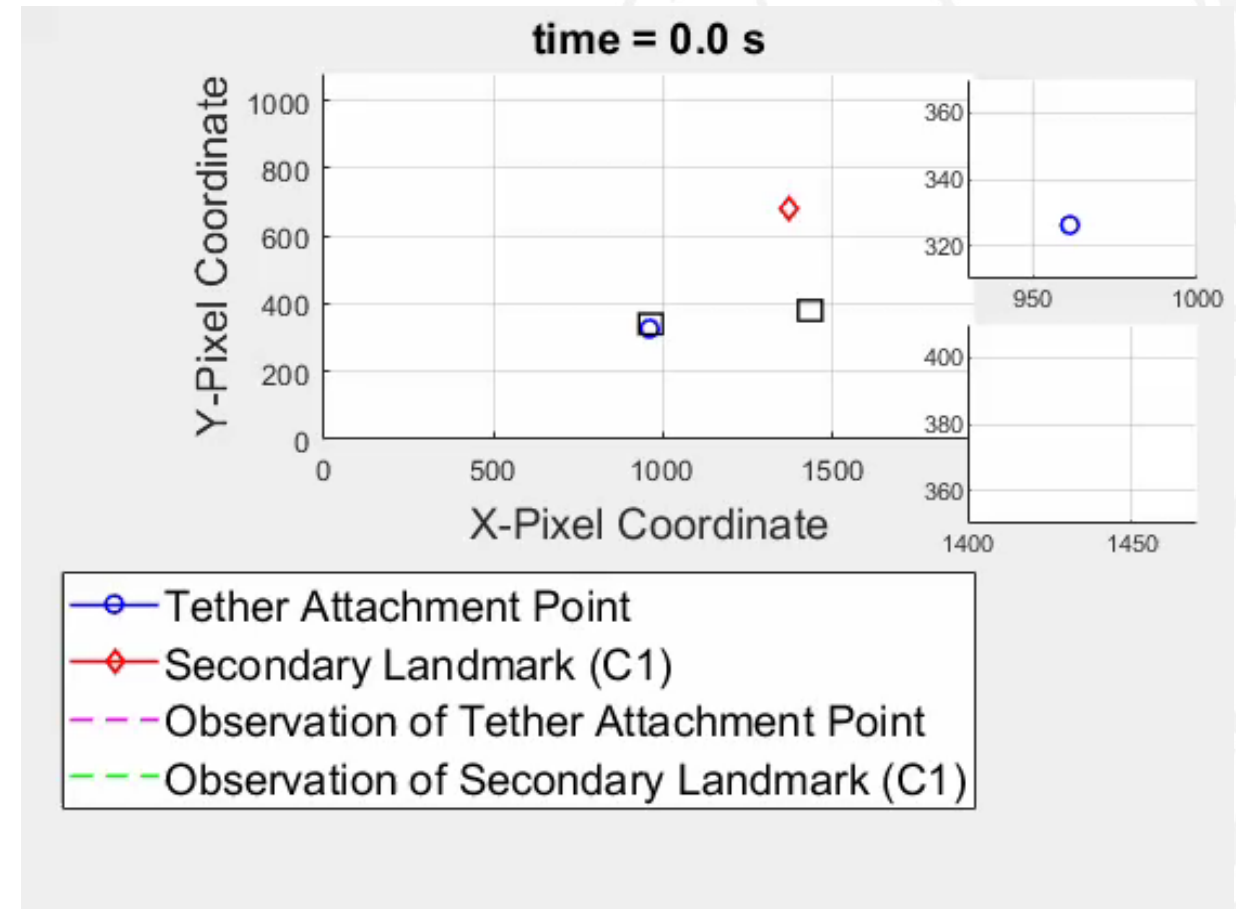
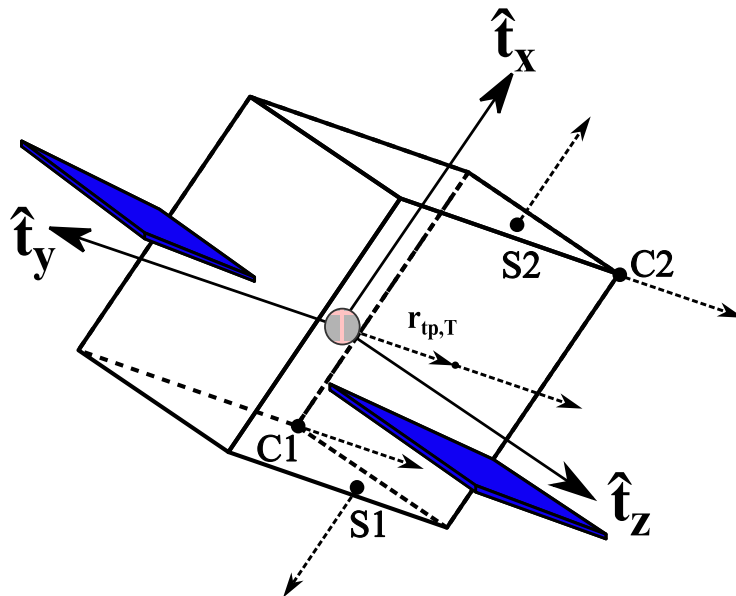
# Camera-Tracked Debris Features

- 5 tracked features
  - 3 on  $-y$  face of debris
  - 1 each on  $+/-x$  face of debris



# Camera Model w/ Occlusion

- Assume known tether attachment points in respective body frames
- Camera generates pixel coordinate measurements of various debris features



# Unscented Kalman Filter Dynamics

- Unscented Quaternion Estimator UKF, States:

$$\mathbf{X} = [\delta p_x, \delta p_y, \delta p_z, \omega_x, \omega_y, \omega_z, J_x, J_y, J_z]^T$$

- Measurement model utilizes tension and pixel coordinate measurements ( $\mathbf{h}(\mathbf{x}) = [T, p_x, p_y]^T$ )
- Measured tension used directly in dynamics

$$\begin{bmatrix} \dot{\hat{q}}_1 \\ \dot{\hat{q}}_2 \\ \dot{\hat{q}}_3 \\ \dot{\hat{q}}_4 \\ \dot{\hat{\omega}}_x \\ \dot{\hat{\omega}}_y \\ \dot{\hat{\omega}}_z \\ \dot{\hat{J}}_x \\ \dot{\hat{J}}_y \\ \dot{\hat{J}}_z \end{bmatrix} = \begin{bmatrix} (\hat{q}_4 \hat{\omega}_x - \hat{q}_3 \hat{\omega}_y + \hat{q}_2 \hat{\omega}_z)/2 \\ (\hat{q}_3 \hat{\omega}_x + \hat{q}_4 \hat{\omega}_y - \hat{q}_1 \hat{\omega}_z)/2 \\ (\hat{q}_1 \hat{\omega}_y - \hat{q}_2 \hat{\omega}_x + \hat{q}_4 \hat{\omega}_z)/2 \\ (-\hat{q}_1 \hat{\omega}_x - \hat{q}_2 \hat{\omega}_y - \hat{q}_3 \hat{\omega}_z)/2 \\ (r_y \hat{T}_z - r_z \hat{T}_y - \hat{\omega}_y \hat{J}_z \hat{\omega}_z + \hat{\omega}_z \hat{J}_y \hat{\omega}_y)/\hat{J}_x \\ (r_z \hat{T}_x - r_x \hat{T}_z - \hat{\omega}_z \hat{J}_x \hat{\omega}_x + \hat{\omega}_x \hat{J}_z \hat{\omega}_z)/\hat{J}_y \\ (r_x \hat{T}_y - r_y \hat{T}_x - \hat{\omega}_x \hat{J}_y \hat{\omega}_y + \hat{\omega}_y \hat{J}_x \hat{\omega}_x)/\hat{J}_z \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

# Relative Distance Control

- Proportional-Integrative-Derivative Controller

- Reach and maintain a desired elongation,  $\Delta l$

- Process variable:  $\hat{e}_k = \Delta l + l_0 - \hat{l}_k^-$

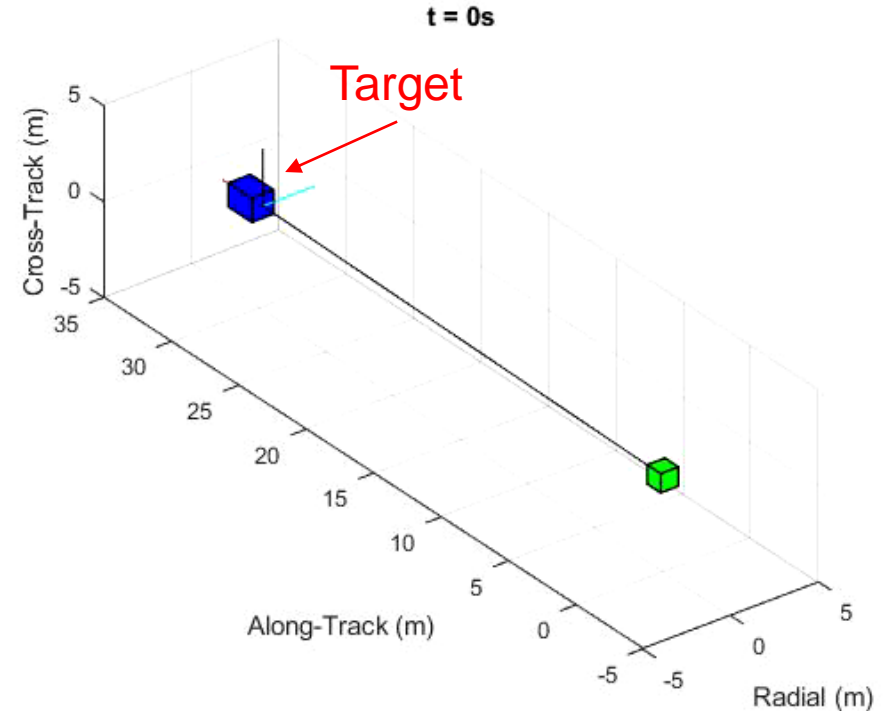
$$\hat{l}_k^- = \|\mathbf{R}_C + {}^{\mathcal{O}}\mathbf{r}_{tp,C} - \mathbf{R}_T - {}^{\mathcal{O}}\hat{\mathbf{r}}_{tp,T}\|$$

- Determines thrust:

$$\mathbf{U} = - \left( K_P \hat{e}_k + K_I \int_0^t \hat{e}_k dt + K_D \dot{\hat{e}}_k \right) \frac{\mathbf{V}_C}{\|\mathbf{V}_C\|}$$

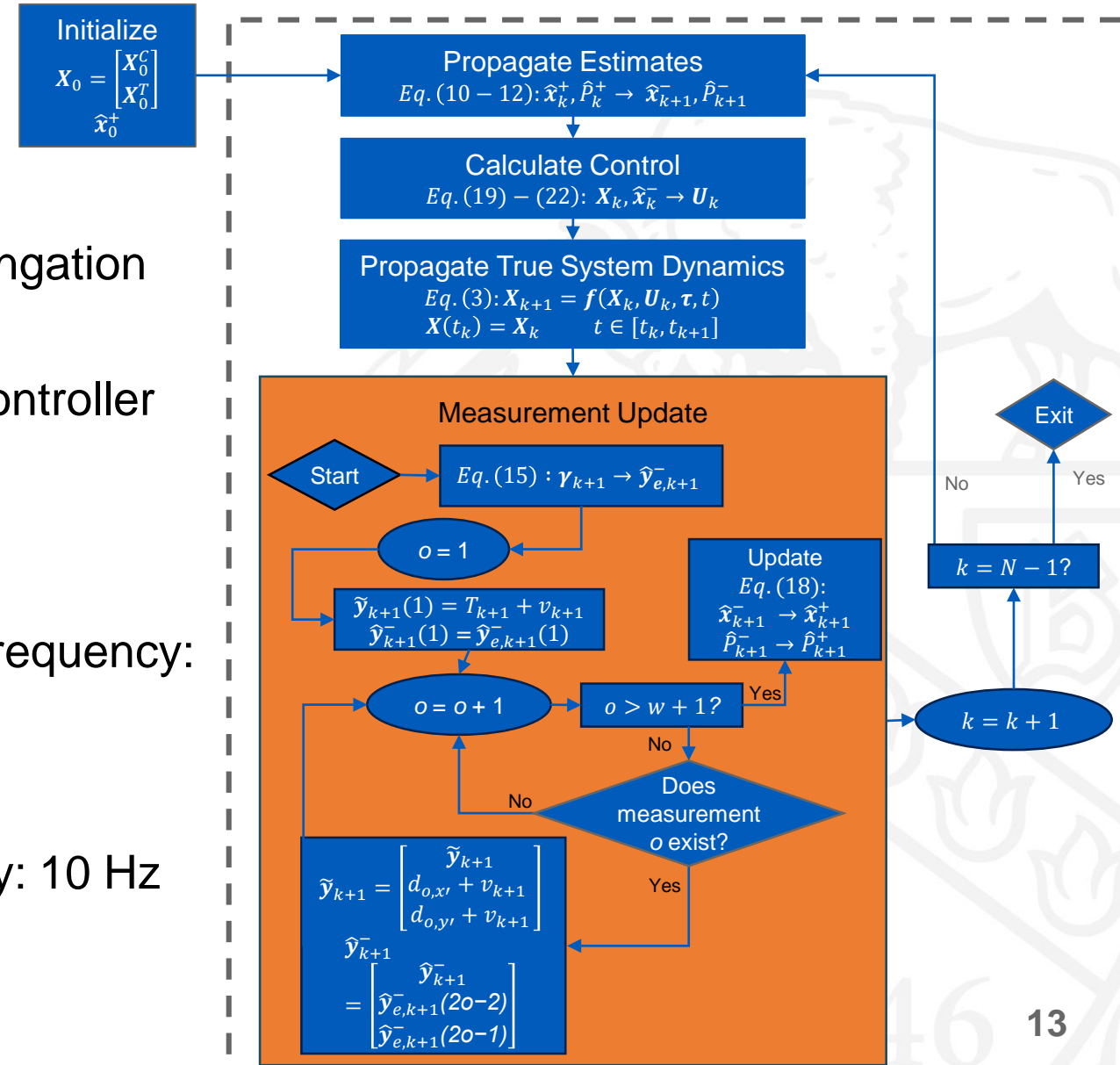
- Uses estimated debris states and parameters:  $\dot{\hat{e}}_k = -\dot{\hat{l}}_k^-$

$$\dot{\hat{l}}_k^- = (\mathbf{V}_C + {}^{\mathcal{O}}\mathbf{A}^c \boldsymbol{\omega}_C \times^c \mathbf{r}_{tp,C} - \mathbf{V}_T - {}^{\mathcal{O}}\hat{\mathbf{A}}^T \hat{\boldsymbol{\omega}}_T \times^T \mathbf{r}_{tp,T}) \cdot \frac{\mathbf{V}_C}{\|\mathbf{V}_C\|}$$



# Control Diagram

- Control
  - Reach and maintain desired elongation
  - 50 N saturation
  - A priori* estimates used in PID controller
- 500 Monte-Carlo runs
  - 2 cases camera measurement frequency:
    - 1 Hz
    - 10 Hz
  - Tension measurement frequency: 10 Hz





# Simulation Parameters

Parameter	Value
Chaser Inertia Matrix $J_C$ (kg-m <sup>2</sup> )	diag(83.3, 83.3, 83.3)
Target Inertia Matrix $J_T$ (kg-m <sup>2</sup> )	diag(15000, 3000, 15000)
Chaser Mass $m_C$ (kg)	500
Target Mass $m_T$ (kg)	3000
Tether Young's Modulus $E$ (Pa)	$60 \times 10^9$
Tether Diameter $d$ (m)	0.001
Tether Natural Length $l_0$ (m)	30
Tether Damping $c$ (Ns/m)	16
Tether Attachment Point, Chaser $\mathbf{r}_{tp,C}$ (m)	$[0.5, 0, 0]^T$
Tether Attachment Point, Target $\mathbf{r}_{tp,T}$ (m)	$[0, -0.875, 0]^T$



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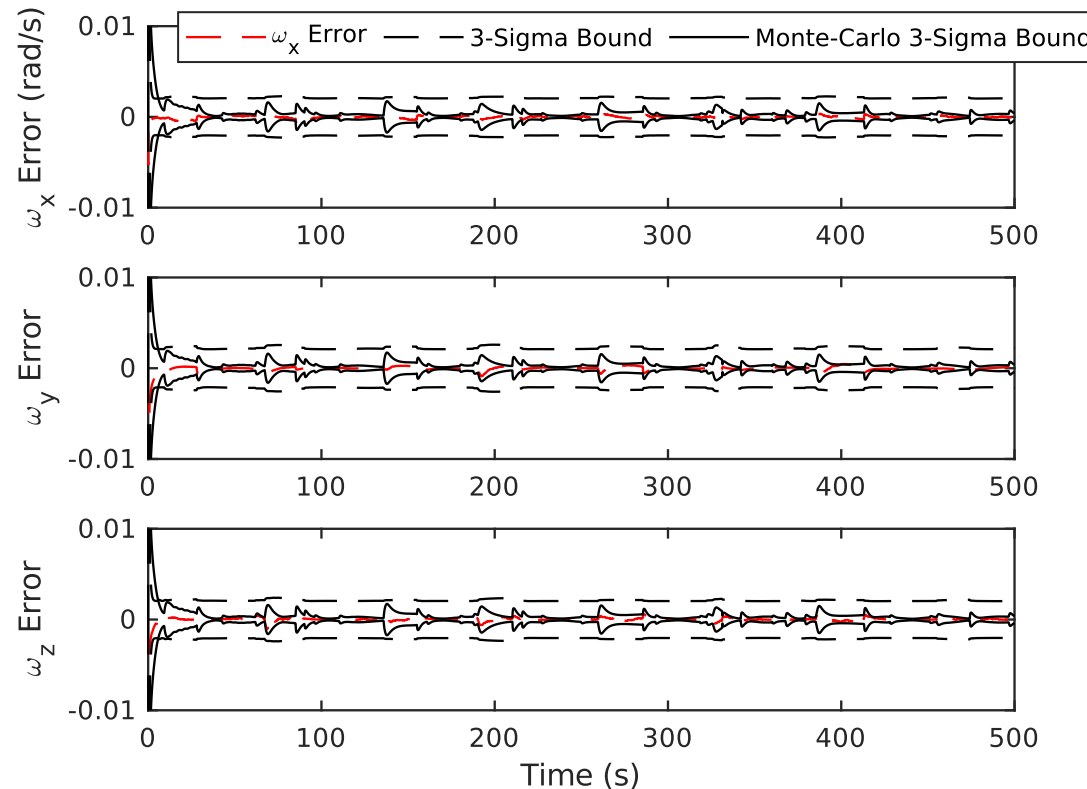
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## Results: Angular Velocity

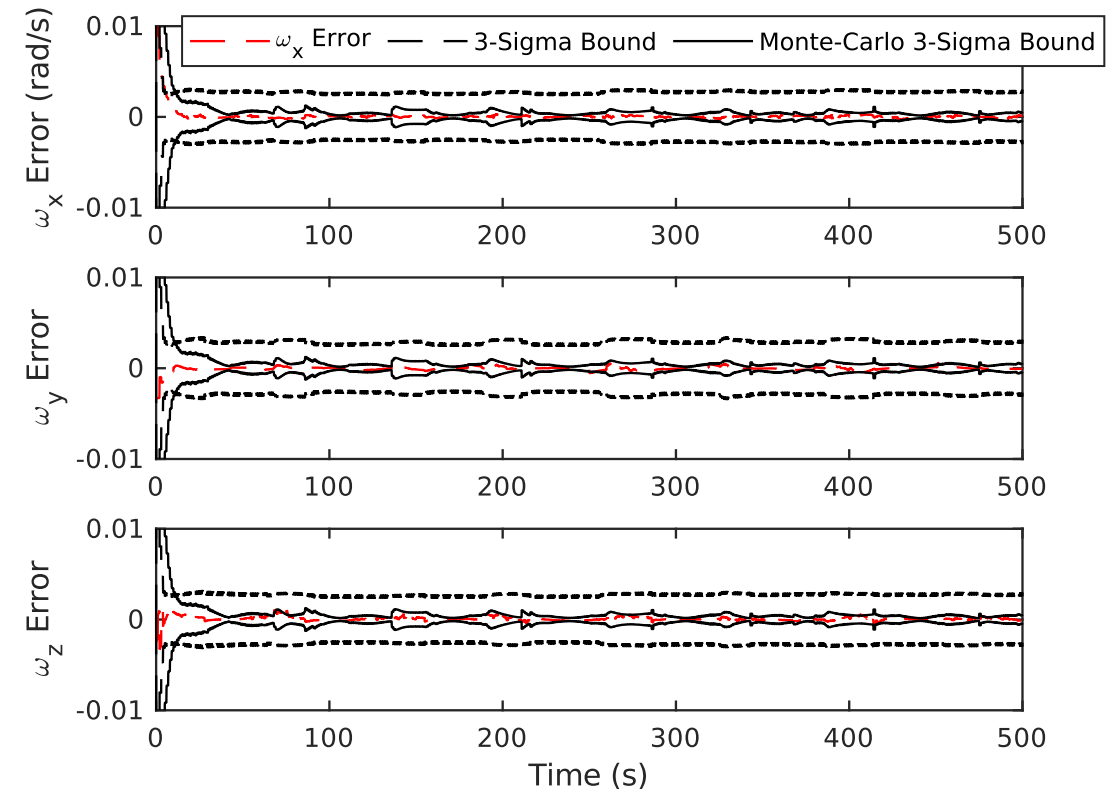
Accurate estimates in both cases

10 Hz



1 Hz causes slightly longer convergence time

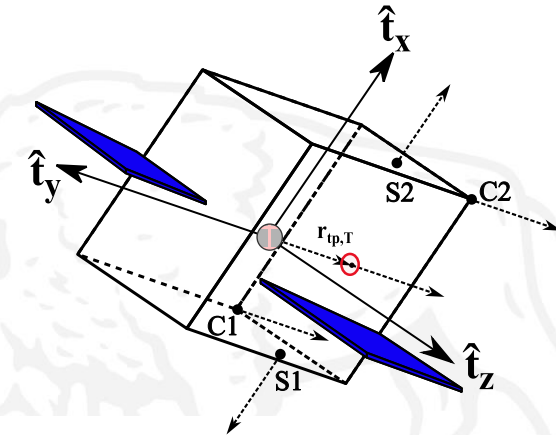
1 Hz



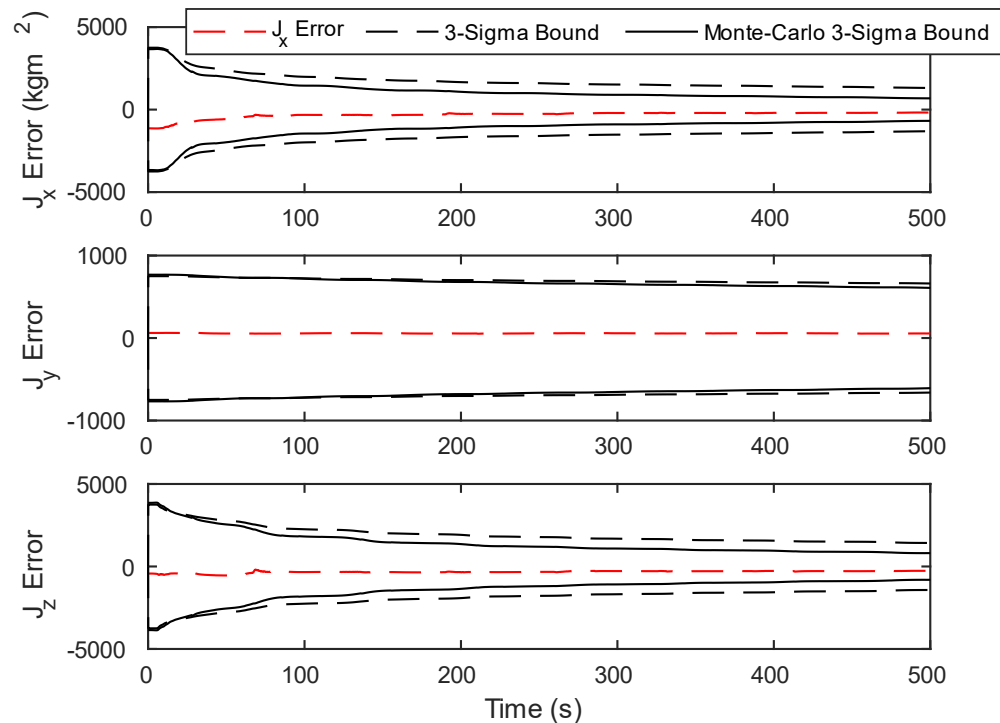
## Results: Principal MMI Estimates

1 Hz:

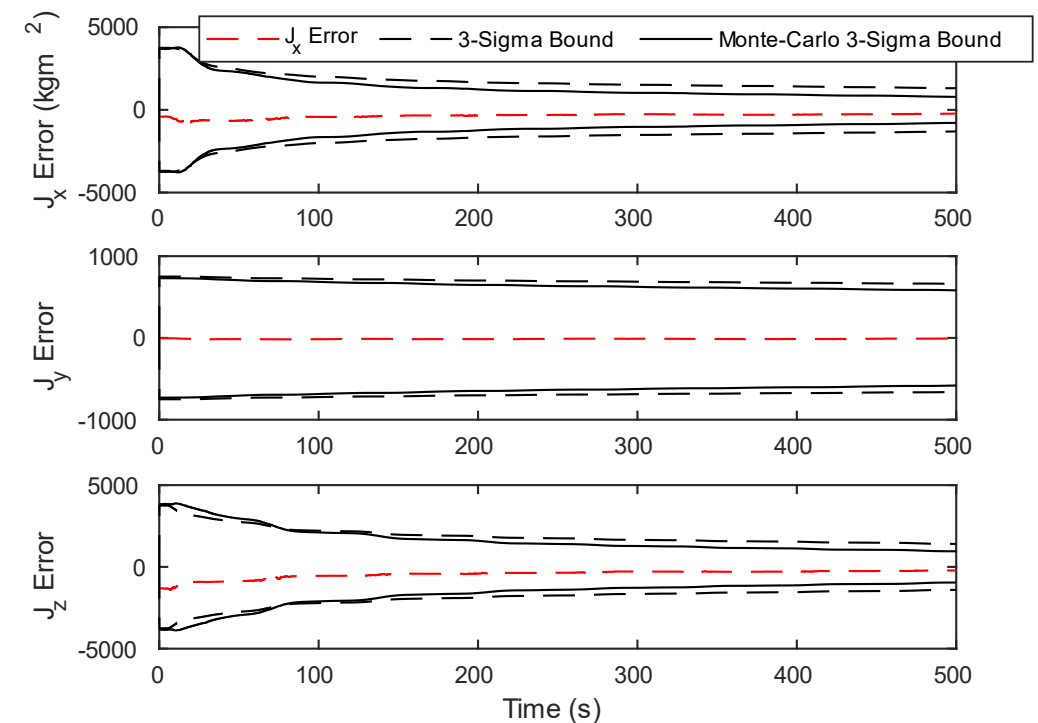
- slightly longer convergence time
- uncertainty more closely matches statistics



10 Hz



1 Hz



## Results: Final Principal MMI Estimates

Parameters	$J_x$	$J_y$	$J_z$
True Value (kg-m <sup>2</sup> )	15000	3000	15000

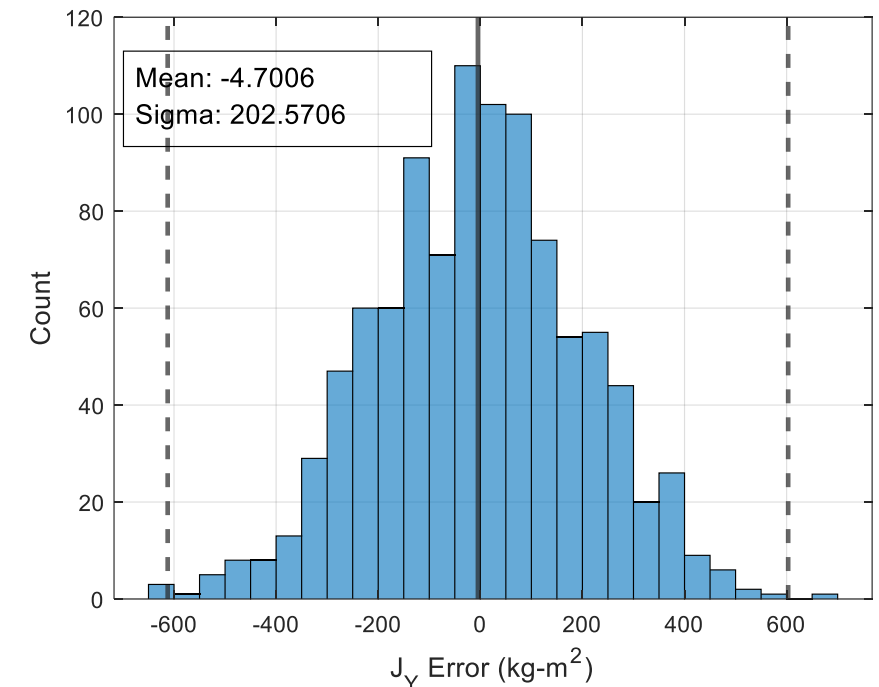
10 Hz

Parameters	$J_x$	$J_y$	$J_z$
Mean	0.23%	-0.15%	0.29%
STD	1.51%	6.75%	1.79%
99.73% Final Estimates	4.76%	20.40%	5.66%

1 Hz

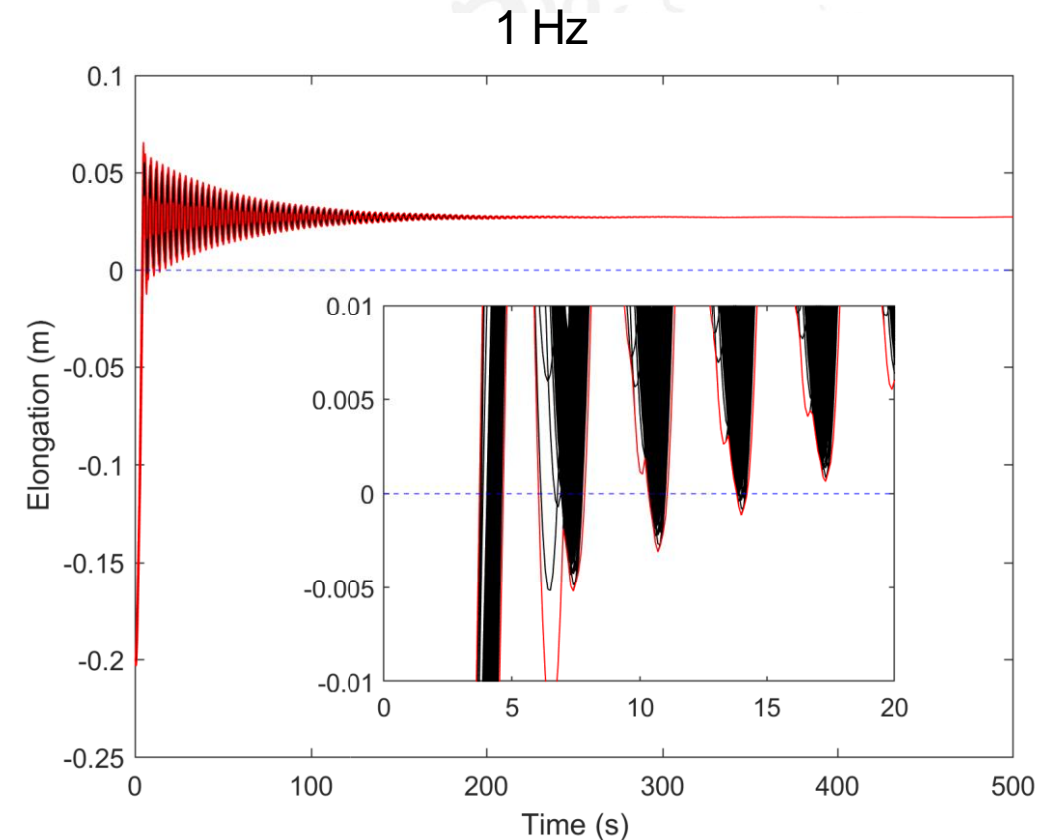
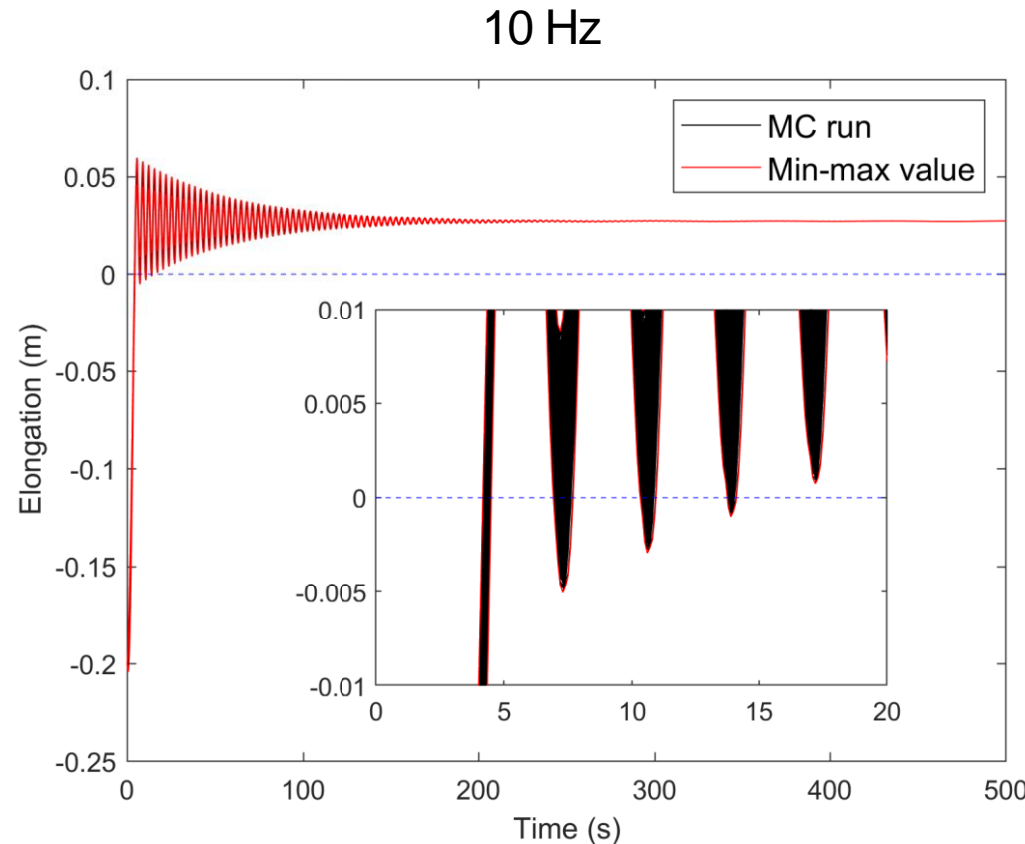
Parameters	$J_x$	$J_y$	$J_z$
Mean	0.18%	-0.41%	0.21%
STD	1.75%	6.46%	2.12%
99.73% Final Estimates	5.43%	19.79%	6.57%

- Update frequency effect on final estimates is negligible
- Final estimate distribution is **normal**



## Results: Elongation

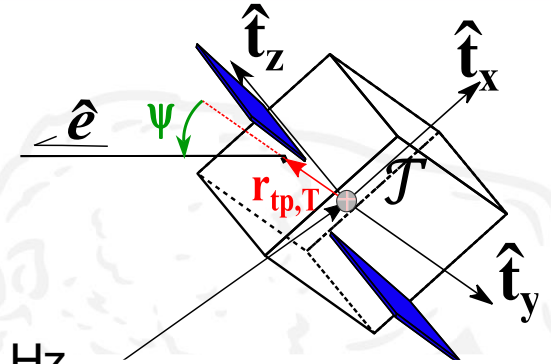
Control objective: Maintain safe post-capture operations



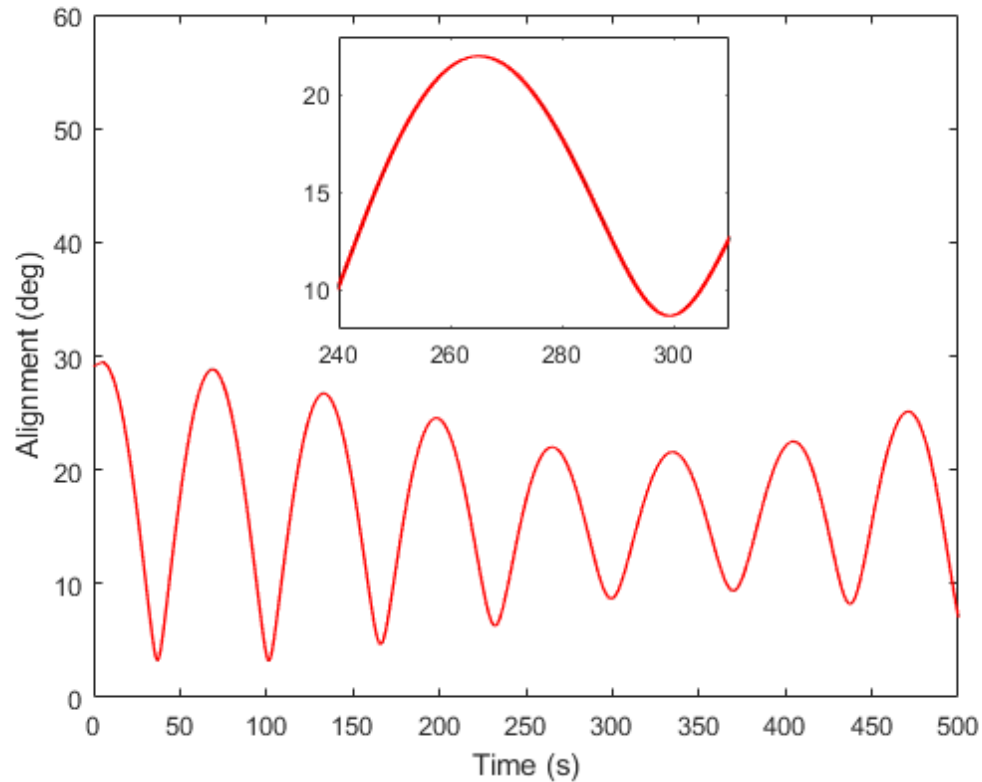
1 Hz has greater deviation prior to settling

## Results: Alignment Angle

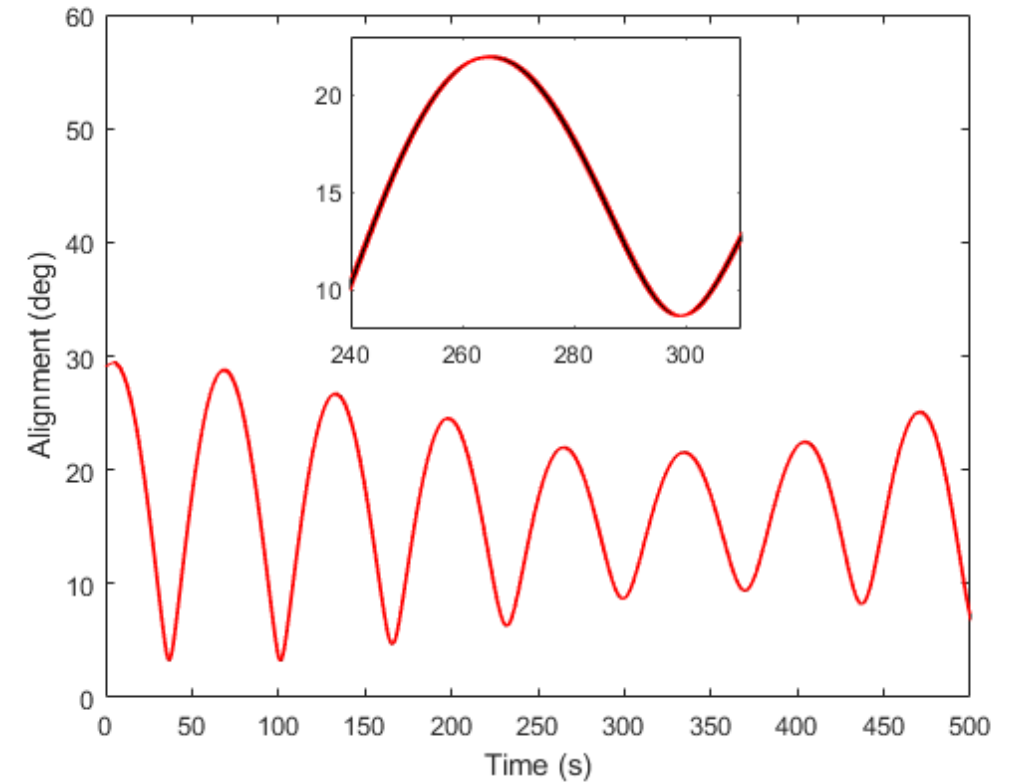
Safety is maintained (**alignment angle  $\Psi < 90^\circ$** )



10 Hz



1 Hz



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# Conclusion

- Separately developed control and estimation algorithms are now implemented together.
- Two different camera measurement frequencies investigated (1 Hz, 10 Hz).
- Estimation performance is minorly affected by frequency of measurements.
- Safe post-capture operations maintained with both measurement frequencies.

## Future work

- Release additional required assumptions (e.g., known center of mass position)
- Modify controller type and control saturation limit

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# References

1. Field, L., Botta, E.M. "Relative Distance Control of Uncooperative Tethered Debris." J Astronaut Sci 70, 55 (2023). <https://doi.org/10.1007/s40295-023-00422-7>
2. Bourabah, D., Gnam, C., and Botta, E.M. "Inertia tensor estimation of tethered debris through tether tracking." Acta Astronautica 212 (2023) <https://doi.org/10.1016/j.actaastro.2023.08.021>

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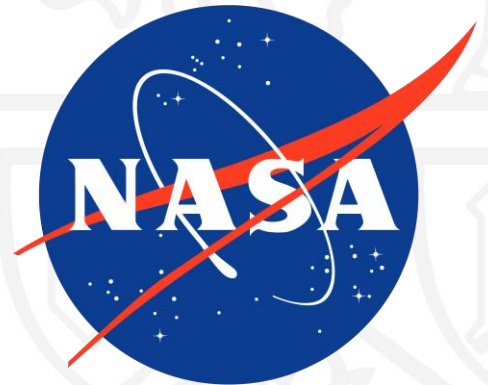
# Thank you !

Questions?:

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