Estimation of Uncooperative Space Debris Inertial Parameters after Tether Capture

Derek Bourabah*, **Liam Field**, **Eleonora Botta**

3rd IAA Conference on Space Situational Awareness
GMV, Madrid, Spain

April 4-6

University at Buffalo The State University of New York

Overview

- Introduction and Motivation
- Data Generation
    - System Model
    - Simulation and Control of Tethered System
    - Data Generation
- Kalman Filtering
    - Filter Methodology
    - Estimation Results
- Conclusions

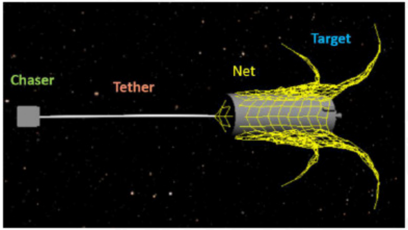
Overview

Introduction and Motivation

Tether-based Capture of Debris

- Promising methods for ADR include tether-based capture (harpoons, tether nets)
- Controls for de-tumbling, attitude, collision prevention, etc.
  - Requires understanding of debris tumbling motion
- Tumbling motion governed by debris moments of inertia

- Principal Moments of Inertia Estimation
  - Necessary for proper implementation of control algorithms
  - Most works find ratios



(Botta et al., 2020)

Introduction and Motivation

Challenges

- Observability
  - Knowledge of acting moments required
    - Only inertia parameter ratios may be found otherwise
    - Tether tension creates a known moment
  - Tether may become slack after capture
    - Loss of observability due to lack of moment knowledge
- Estimation
  - Implement EKF and UKF
    - UKF serves as a benchmark for EKF comparison
    - Determines how well the parameters may actually be estimated
  - Investigate 2 cases of tether state
    - Frequently slack
    - Frequently taut

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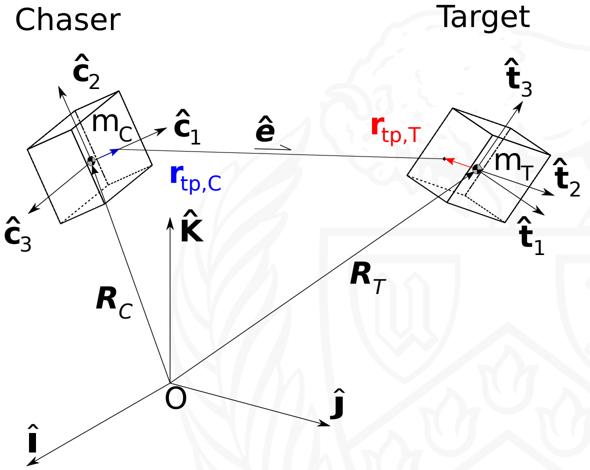
Data Generation System Model

- Rigid body chaser/ rigid body debris connected by massless, extensible tether modeled as single spring-damper

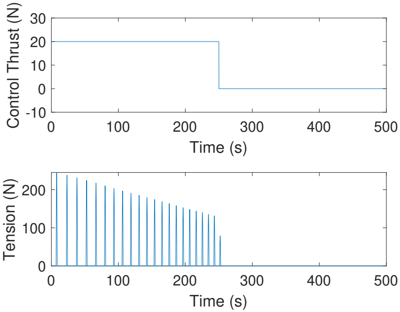
  $[\hat{\boldsymbol{I}}, \hat{\boldsymbol{J}}, \hat{\boldsymbol{K}}]$: ECI Frame
  $[\hat{\boldsymbol{c}}_1, \hat{\boldsymbol{c}}_2, \hat{\boldsymbol{c}}_3]$: Chaser body frame
  $[\hat{\boldsymbol{t}}_1, \hat{\boldsymbol{t}}_2, \hat{\boldsymbol{t}}_3]$ or $[\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}]$: Target body frame
  $\boldsymbol{r}_{tp}$: Tether attachment point



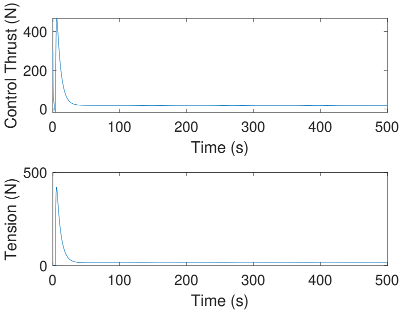
Model Simulation and Control

- Full dynamics numerically simulated
- Attitude of chaser controlled via sliding mode control
- Two controls maintain distance between chaser/debris, preventing collision post-capture

Case 1: Open-loop Control



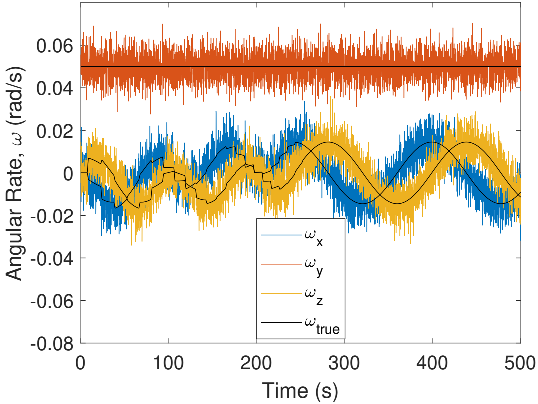
Case 2: PID Control



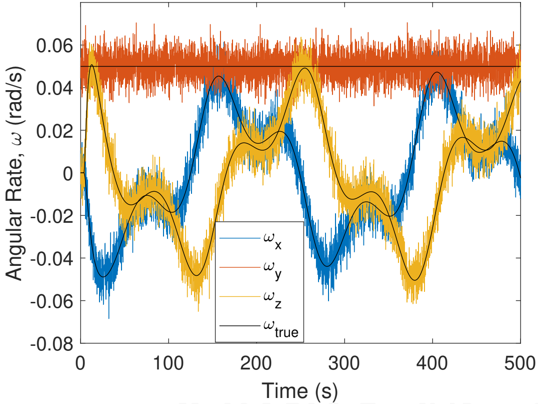
- Brief periods of tension (~2 s)
- Large initial tension, constantly in tension

Data Generation Example Measurements

Case 1



Case 2



- Full dynamics simulated to generate true angular rates
- Noise added to true angular rates to simulate measurements, $\sim N(0{,}0.345)$ deg/s

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Kalman Filter Dynamics

Principle Moments of Inertia Estimation

- Assume tether attachment point in body frame and tether tension are known perfectly
- Due to noisy measurements, true angular rate must be an estimated state
- Output: target angular rates alone ($\boldsymbol{h}(\boldsymbol{X}) = [\omega_x, \omega_y, \omega_z]^T$)
- State: $\boldsymbol{X} = [\omega_x, \omega_y, \omega_z, J_x, J_y, J_z]^T$

$$\dot{\boldsymbol{X}} = \mathbf{f}(\boldsymbol{X}(t), \boldsymbol{U}(t), t) = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \dot{J}_x \\ \dot{J}_y \\ \dot{J}_z \end{bmatrix} = \begin{bmatrix} (r_y T_z - r_z T_y - \omega_y J_z \omega_z + \omega_z J_y \omega_y)/J_x \\ (r_z T_x - r_x T_z - \omega_z J_x \omega_x + \omega_x J_z \omega_z)/J_y \\ (r_x T_y - r_y T_x - \omega_x J_y \omega_y + \omega_y J_x \omega_x)/J_z \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

EKF Sensitivity Matrix Modification

- Standard UKF, modified EKF
- Commonly, only $\dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}(\boldsymbol{J})$
- Assume angular rate is a function of inertia properties, $\boldsymbol{\omega} = \boldsymbol{\omega}(\boldsymbol{J})$

Sensitivity matrix becomes:

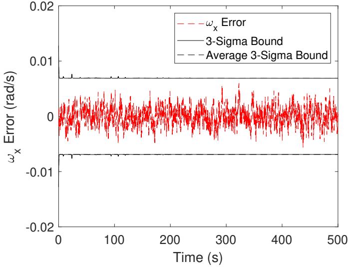
$$F(\hat{\boldsymbol{X}}_k^+, t) \equiv \frac{\partial \mathbf{f}}{\partial \boldsymbol{X}}\bigg|_{\hat{\boldsymbol{X}}_k^+} = \begin{bmatrix} \frac{\partial \dot{\boldsymbol{\omega}}}{\partial \boldsymbol{\omega}}\Big|_{\hat{\boldsymbol{X}}_k^+} & \frac{\partial \dot{\boldsymbol{\omega}}}{\partial \boldsymbol{J}}\Big|_{\hat{\boldsymbol{X}}_k^+} \\ 0_{3x3} & 0_{3x3} \end{bmatrix}$$

Angular Rate Estimation

- Monte-Carlo simulation (1000 runs) to test performance of filters
- Angular rate 3σ bounds across all Monte-Carlo runs are similar for each respective filter
    - Tether state does not affect estimation of angular rates

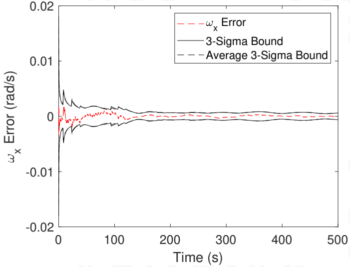
Sample ω errors for Case 1

EKF



3σ bounds $\sim$ 0.006 rad/s

UKF



3σ bounds $\sim$ 0.0007 rad/s

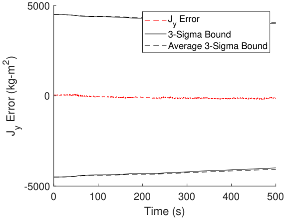
J Estimation - Case 1: Frequently Slack Tether

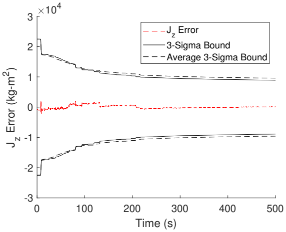
EKF

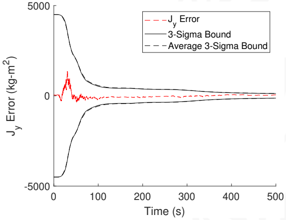
- Unable to confidently converge to an estimate for J_y
- Rapid convergence in confidence during tension events

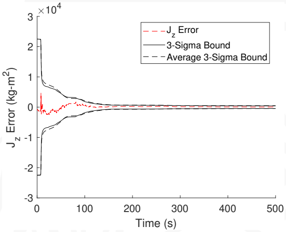
UKF

- Estimates J_y well
- Significantly more accurate than the EKF
- Convergence of 3σ bounds begins at first instance of tether becoming taut









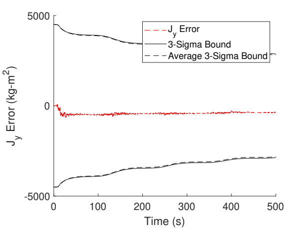
J Estimation - Case 2: Frequently Taut Tether

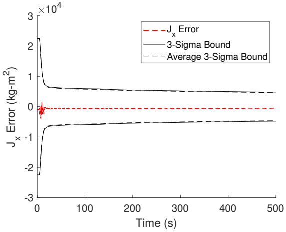
EKF

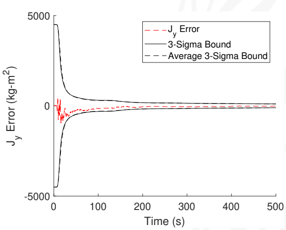
- Slow convergence of J_y 3σ bounds
- J_x and J_z 3σ bounds rapidly converge, then slowly improve
- Large 3σ bound for final estimate

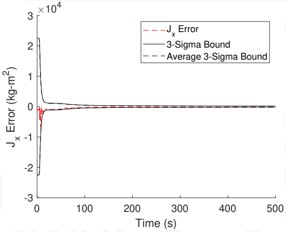
UKF

- Significantly more accurate, average errors < 10 kg-m^2
- Rapidly converges to true $\boldsymbol{J}$ within 150 s



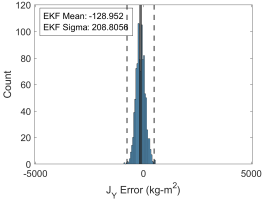


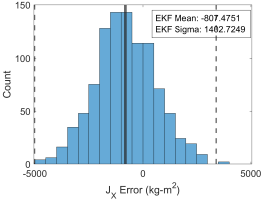




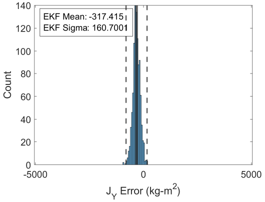
EKF Final Estimate Error Distribution

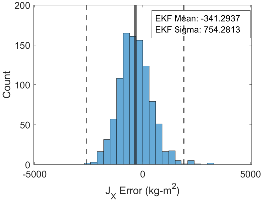
Case 1





Case 2





$\boldsymbol{J_y}$

- Difficult for EKF to estimate
- Average error estimate worsens Case 1 $\rightarrow$ Case 2
- 3σ bound becomes smaller

$\boldsymbol{J_x}$

- Significant improvement in average error
- 3σ bounds are halved 1402 kg-m^2 $\rightarrow$ 754 kg-m^2

UKF Final Estimate Error Distribution

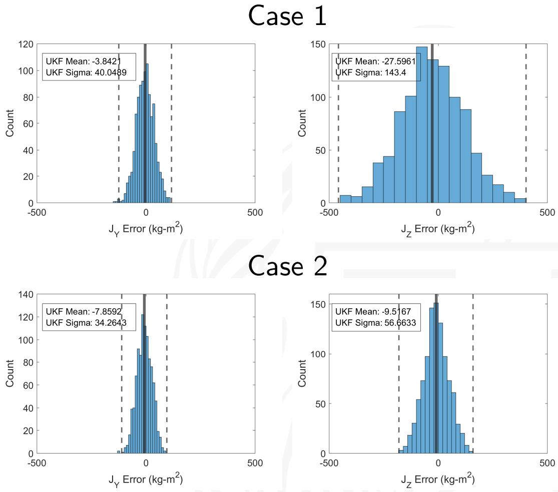
More precise and accurate than EKF

$\boldsymbol{J_y}$

- Case 1 $\rightarrow$ Case 2 3σ bounds decrease, average error increase
- Closer estimate than EKF, however unobservability is still evident

$\boldsymbol{J_z}$

- Improvements in both average error and 3σ bound Case 1 $\rightarrow$ Case 2



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Conclusions

- First work on **estimation of all three principal moments of inertia on tethered debris**
- Assumptions:
  - Perfect knowledge of tension
  - Noisy angular rate measurements
  - Tether attachment point is known
- **Knowledge of tension in the tether and angular rate of the target is sufficient** to estimate the principal moment of inertia values
  - UKF: performs very well with **high precision and accuracy**
  - EKF: **too uncertain** to properly estimate the moment of inertia values
- **Angular rate estimates** unaffected by tether slackness
- Despite frequent tether slackness, the **principal moments of inertia may still be estimated**

Questions?

derekbou@buffalo.edu
ebotta@buffalo.edu

Funded by the National Science Foundation
Award# CRII: RI:2105011

NASA Space Grant Fellowship